

NUS-RMI Credit Rating Initiative Technical Report

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RMI staff article

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INTRODUCTION

This document describes the implementation of the system which the NUS Risk Management Institute's Credit Rating Initiative uses to produce probabilities of default (PDs). As of this version of the Technical Report, these PDs cover exchange listed firms in 30 economies in Asia, Asia-Pacific, North America and Western Europe. The individual PDs for nearly 30,000 firms are computed daily. 2,200 of these firms' default forecasts are freely available to all users at www.rmi.nus.edu.sg/cri, along with aggregate PDs at the economy and sector level for all the firms.

The primary goal of this initiative is to drive research and development in the critical area of credit rating systems. As such, a transparent methodology is essential to this initiative. Having the details of the methodology available to everybody means that there is a base from which suggestions and improvements can be made. The objective of this Technical Report is to provide a full exposition of the CRI system. Readers of this document

who have access to the necessary data and who have a sufficient level of technical expertise will be able to implement a similar system on their own.

The system used by the CRI will evolve as new innovations and enhancements are applied. This Technical Report will be updated to reflect changes in the system. All versions will be available via the web portal.

The remainder of this Technical Report is organized as follows. The next section describes the quantitative model that is currently used to compute PDs from the CRI. The model was first described in Duan, Sun and Wang (2011). The description includes calibration procedures, which are performed on a monthly basis, and individual firm PD computations, which are performed on a daily basis.

Section 2 will describe the input variables of the model as well as the data used to produce the variables for input into the model. This model uses both input variables that are common to all firms in an economy and input variables that are firm-specific. Another critical component

when calibrating a credit rating system is the default data, and this is also described in this section.

While Section 1 provides a broader description of the model, Section 3 will describe the implementation details that are necessary to apply given real world issues of, for example, bad or missing data. The specific technical details needed to develop an operational system are also given, including details on the monthly calibration, daily computation of individual firm PDs and aggregation of the individual firm PDs. Distance-to-default (DTD) in a Merton-type model is one of the firm-specific variables. The calculation for DTD is not the standard one, and has been modified to allow a meaningful computation of the DTD for financial firms. While most academic studies on default prediction exclude financial firms from consideration, it is important to include them given that the financial sector is a critical component in every economy. The calculation for DTD is detailed in this section.

Section 4 shows an empirical analysis for those economies that are currently covered. While the analysis shows excellent results in several economies, there is room for improvement in a few others. This is because, at the CRI's current stage of development the economies all use the variables used in the academic study of US firms in Duan et al. (2011). Future development within the CRI will deal with variable selection specific to different economies, and the performance is then expected to improve. Variable selection and other planned developments are discussed in Section 5.

I. MODEL DESCRIPTION

The quantitative model that is currently being used by the CRI is a forward intensity model that was introduced in Duan et al. (2011). This model allows default forecasts to be made at a range of horizons. In the current CRI implementation of this model, PDs are computed from a horizon of one month up to a horizon of two years. In other words, for every firm, the probability of that firm defaulting within one month, three months, six months, one year, eighteen months and two years is given. The ability to assess credit quality for different horizons

is a useful tool for risk management, credit portfolio management, policy setting and regulatory purposes, since short and long-term credit risk profiles can differ greatly depending on a firm's liquidity, debt structure and other factors.

The forward intensity model is a reduced form model in which the probability of default is computed as a function of different input variables. These can be firm-specific or common to all firms within an economy. The other category of default prediction model is the structural model, whereby the corporate structure of a firm is modeled in order to assess the firm's probability of default.

A similar reduced form model by Duffie, Saita and Wang (2007) relied on modeling the time series dynamics of the input variables in order to calculate PDs for different horizons. However, there is little consensus on assumptions for the dynamics of variables such as accounting ratios, and the model output will be highly dependent on these assumptions. In addition, the time series dynamics will be of very high dimension. For example, with the two common variables and two firm-specific variables that Duffie et al. (2007) use, a sample of 10,000 firms gives a dimension of the state variables of 20,002.

Given the complexity in modeling the dynamics of variables such as accounting ratios, this model will be difficult to implement if different forecast horizons are required. The key innovation of the forward intensity model is that PD for different horizons can be consistently and efficiently computed based only on the value of the input variables at the time the prediction is made. Thus, the model specification becomes far more tractable.

Fully specifying a reduced form model includes the specification of the function that computes a PD from the input variables. This function is parameterized, and finding appropriate parameter values is called *calibrating* the model. The forward intensity model can be calibrated by maximizing a pseudo-likelihood function. The calibration is carried out by economy and all firms within an economy will use the same parameter values along with each firms' variables in order to compute a firm's PD.

Subsection 1.1 will describe the modeling framework, including the way PDs are computed based on a set of parameter values for the economy and a set of input variables for a firm. Subsection 1.2 explains how the model can be calibrated.

1.1 Modeling Framework

While the model can be formulated in a continuous time framework, as done in Duan et al. (2011), an operational implementation will require discretization in time. Since the model is more easily understood in discrete time, the following exposition of the model will begin in a discrete time framework.

Variables for default prediction can have vastly different update frequencies. Financial statement data is updated only once a quarter or even once a year, while market data like stock prices are available at frequencies of seconds. A way of compromising between these two extremes is to have a fundamental time period Δt of one month in the modeling framework. As will be seen later, this does not preclude updating the PDs on a daily basis. This is important since, for example, large daily changes in a firm's stock price can signal changes in credit quality even when there is no change in financial statement data.

Thus, for the purposes of calibration and subsequently for computing time series of PD, the input variables at the end of each month will be kept for each firm. The input variables associated with the i^{th} firm at the end of the n^{th} month (at time $t = n\Delta t$) is denoted by $X_i(n)$. This is a vector consisting of two parts: $X_i(n) = (W(n), U_i(n))$. Here, $W(n)$ is a vector of variables at the end of month n that is common to all firms in the economy and $U_i(n)$ is a vector of variables specific to firm i .

In the forward intensity model, a firm's default is signaled by a jump in a Poisson process. The probability of a jump in the Poisson process is determined by the *intensity* of the Poisson process. The forward intensity model draws an explicit dependence of intensities at time periods in the future (that is, forward intensities) to the value of input variables at the time of prediction. With forward intensities, PDs for any horizon can be

computed knowing only the value of the input variable at the time of prediction, without needing to simulate future values of the input variables.

There is a direct analogy in interest rate modeling. In spot rate models where dynamics on a short-term spot rate are specified, bond pricing requires expectations on realizations of the short rate. Alternatively, bond prices can be computed directly if the forward rate curve is known.

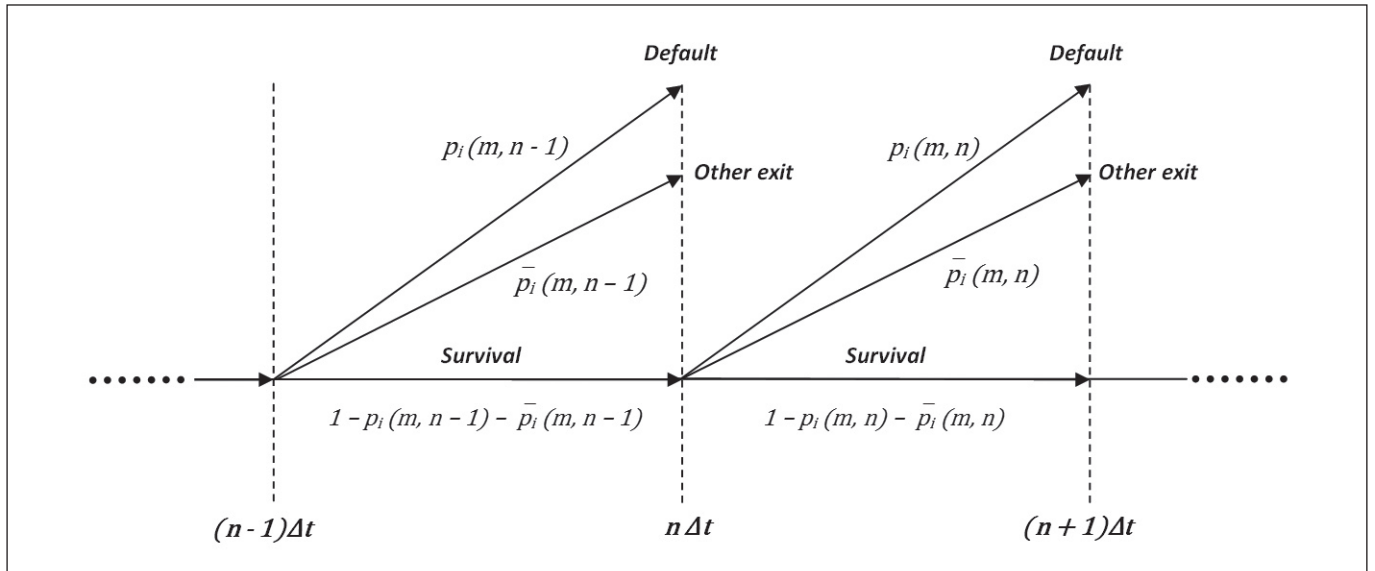
One issue in default prediction is that firms can exit public exchanges for reasons other than default. For example, in mergers and acquisitions involving two public companies, there will be one company that delists from its stock exchange. This is important in predicting defaults because a default cannot happen if a firm has been previously delisted. An exception is if the exit is a distressed exit and is followed soon after by a credit event. See Subsection 2.4 for details on how this case is handled in the CRI system.

In order to take these other exits into account, defaults and other exits are modeled as two independent Poisson processes, each with their own intensity. While defaults and exits classified as non-defaults are mutually exclusive by definition, the assumption of independent Poisson processes does not pose a problem since the probability of a simultaneous jump in the two Poisson processes is negligible. In the discrete time framework, the probability of simultaneous jumps in the same time interval is non-zero. As a modeling assumption, a simultaneous jump in the same time interval by both the default Poisson process and the non-default type exit Poisson process is considered as a default. In this way, there are three mutually exclusive possibilities during each time interval: survival, default and non-default exit. As with defaults, the forward intensity of the Poisson process for other exits is a function of the input variables. The parameters of this function can also be calibrated.

To further illustrate the discrete framework, the three possibilities for a firm at each time point are diagrammed. Either the firm survives for the next time period Δt , or it defaults within Δt , or it has a non-default exit within Δt . This setup is pictured in Figure 1. Information about firm i is known up until time $t = m\Delta t$ and the figure illustrates possibilities in the future

FIGURE 1

Default-other exit-survival tree for firm i , viewed from time $t = m\Delta t$.



between $t = (n - 1) \Delta t$ and $(n + 1) \Delta t$. Here, m and n are integers with $m < n$.

The probabilities of each branch are, for example: $p_i(m, n)$ the conditional probability viewed from $t = m\Delta t$ that firm i will default before $(n + 1) \Delta t$, conditioned on firm i surviving up until $n\Delta t$. Likewise, $\bar{p}_i(m, n)$ is the conditional probability viewed from $t = m\Delta t$ that firm i will have a non-default exit before $(n + 1) \Delta t$, conditioned on firm i surviving up until $n\Delta t$. It is the modeler’s objective to determine $p_i(m, n)$ and $\bar{p}_i(m, n)$, but for now it is assumed that these quantities are known. With the conditional default and other exit probabilities known, the corresponding conditional survival probability of firm i is $1 - p_i(m, n) - \bar{p}_i(m, n)$.

With this diagram in mind, the probability that a particular path will be followed is the product of the conditional probabilities along the path. For example, the probability at time $t = m\Delta t$ of firm i surviving until $(n - 1)\Delta t$ and then defaulting between $(n - 1)\Delta t$ and $n\Delta t$ is:

$$\begin{aligned}
 & Prob_{t=m\Delta t}[\tau_i = n, \tau_i < \bar{\tau}_i] \\
 &= p_i(m, n - 1) \prod_{j=m}^{n-2} [1 - p_i(m, j) - \bar{p}_i(m, j)].
 \end{aligned} \tag{1}$$

Here, τ_i is the default time for firm i measured in units of months, $\bar{\tau}_i$ is the other exit time measured in units of months, and the product is equal to one if there are no terms in the product. The condition $\tau_i < \bar{\tau}_i$ is the requirement that the firm defaults before it has a non-default type of exit. Note that by measuring exits in units of months, if, for example, a default occurs at any time in the interval $((n - 1)\Delta t, n\Delta t]$ then $\tau_i = n$.

Using equation (1), cumulative default probabilities can be computed. At $m\Delta t$ the probability of firm i defaulting at or before $n\Delta t$ and not having an other exit before $t = n\Delta t$ is obtained by taking the sum of all of the paths that lead to default at or before $n\Delta t$:

$$\begin{aligned}
 & Prob_{t=m\Delta t}[m < \tau_i \leq n, \tau_i < \bar{\tau}_i] \\
 &= \sum_{k=m}^{n-1} \left\{ p_i(m, k) \prod_{j=m}^{k-1} [1 - p_i(m, j) - \bar{p}_i(m, j)] \right\}.
 \end{aligned} \tag{2}$$

While it is convenient to derive the probabilities given in equations (1) and (2) in terms of the conditional probabilities, expressions for these in terms of the forward intensities need to be found, since the forward intensities will be functions of the input variables $X_i(m)$. The forward intensity for the default of firm

i that is observed at time $t = m\Delta t$ for the forward time interval from $t = n\Delta t$ to $(n + 1)\Delta t$, is denoted by $h_i(m, n)$ where $m \leq n$. The corresponding forward intensity for a non-default exit is denoted by $\bar{h}_i(m, n)$. Because default is signaled by a jump by a Poisson process, its conditional probability is a simple function of its forward intensity:

$$p_i(m, n) = 1 - \exp[-\Delta t h_i(m, n)]. \quad (3)$$

Since joint jumps in the same time interval are assigned as defaults, the conditional other exit probability needs to take this into account:

$$\begin{aligned} \bar{p}_i(m, n) &= 1 - \exp[-\Delta t \bar{h}_i(m, n)] \\ &\quad - \{1 - \exp[-\Delta t h_i(m, n)]\} \\ &\quad \times \{1 - \exp[-\Delta t \bar{h}_i(m, n)]\} \\ &= \exp[-\Delta t h_i(m, n)] \{1 - \exp[-\Delta t \bar{h}_i(m, n)]\}. \end{aligned} \quad (4)$$

The conditional survival probabilities in equations (1) and (2) are computed as the conditional probability that the firm does not default in the period and the firm does not have a non-default exit either:

$$\begin{aligned} \text{Prob}_{t=m\Delta t}[\tau_i, \bar{\tau}_i > n + 1 | \tau_i, \bar{\tau}_i > n] \\ = \exp\{-[h_i(m, n) + \bar{h}_i(m, n)]\Delta t\}. \end{aligned} \quad (5)$$

It remains to specify the dependence of the forward intensities on the input variables $X_i(m)$. The forward intensities need to be positive so that the conditional probabilities are non-negative. A standard way to impose this constraint is to specify the forward intensities as exponentials of a linear combination of the input variables:

$$\begin{aligned} h_i(m, n) &= \exp[\beta(n - m) \cdot Y_i(m)], \\ \bar{h}_i(m, n) &= \exp[\bar{\beta}(n - m) \cdot Y_i(m)]. \end{aligned} \quad (6)$$

Here, β and $\bar{\beta}$ are coefficient vectors that are functions of the number of months between the observation date and the beginning of the forward period ($n - m$), and $Y_i(m)$ is simply the vector $X_i(m)$ augmented by a preceding unit element: $Y_i(m) = (1, X_i(m))$. The unit element allows the linear combination in the argument of the exponentials in equation (6) to have a non-zero intercept.

In the current implementation of the forward intensity model in the CRI, the maximum horizon is 24 months and there are 12 input variables plus the intercept. So there are 24 sets of each of the coefficient vectors denoted by $\beta(0), \dots, \beta(23)$ and $\bar{\beta}(0), \dots, \bar{\beta}(23)$ and each of these coefficient vectors has 13 elements. While this is a large set of parameters, as will be seen in the next subsection, the calibration is tractable because the parameters for each horizon can be done independently from each other, and the default parameters can be calibrated separately from the other exit parameters.

Before giving the probabilities in (1) and (2) in terms of the forward intensities, a notation is introduced for the forward intensities that makes clear which parameters are needed for the forward intensity in question:

$$H(\beta(n - m), X_i(m)) := \exp[\beta(n - m) \cdot Y_i(m)]. \quad (7)$$

This is the forward default intensity. The corresponding notation for other exit forward intensities is then just $H(\bar{\beta}(n - m), X_i(m))$. So, the probability in (1) is expressed in terms of the forward intensities, using (3) for the conditional default probability and (5) for the conditional survival probability:

$$\begin{aligned} \text{Prob}_{t=m\Delta t}[\tau_i = n, \tau_i < \bar{\tau}_i] \\ = \{1 - \exp[-\Delta t H(\beta(n - 1 - m), X_i(m))]\} \\ \times \prod_{j=m}^{n-2} \exp\{-\Delta t [H(\beta(j - m), X_i(m)) \\ + H(\bar{\beta}(j - m), X_i(m))]\} \\ = \{1 - \exp[-\Delta t H(\beta(n - 1 - m), X_i(m))]\} \\ \times \exp\left\{-\Delta t \sum_{j=m}^{n-2} [H(\beta(j - m), X_i(m)) \\ + H(\bar{\beta}(j - m), X_i(m))]\right\}. \end{aligned} \quad (8)$$

This probability will be relevant in the next subsection during the calibration. The cumulative default probability given in equation (2) in terms of the forward intensities is then:

$$\begin{aligned}
& Prob_{t=m\Delta t}[m < \tau_i \leq n, \tau_i < \bar{\tau}_i] \quad (9) \\
& = \sum_{k=m}^{n-1} \left\{ 1 - \exp[-H(\beta(k-m), X_i(m))] \right\} \\
& \quad \times \exp \left\{ -\Delta t \sum_{j=m}^{k-1} \left[H(\beta(j-m), X_i(m)) \right. \right. \\
& \quad \quad \left. \left. + H(\bar{\beta}(j-m), X_i(m)) \right] \right\}.
\end{aligned}$$

This formula is used to compute the main output of the CRI: an individual firm's PD within various time horizons. The β and $\bar{\beta}$ parameters are obtained when the firm's economy is calibrated, and using those together with the firm's input variables yields the firm's PD.

1.2 Model Calibration

The empirical dataset used for calibration can be described as follows. For the economy as a whole, there are N end of month observations, indexed as $n = 1, \dots, N$. Of course, not all firms will have observations for each of the N months as they may start later than the start of the economy's dataset or they may exit before the end of the economy's dataset. There are a total of I firms in the economy, and they are indexed as $i = 1, \dots, I$. As before, the input variables for the i^{th} firm in the n^{th} month is $X_i(n)$. The set of all observations for all firms is denoted by X .

In addition, the default times τ_i and non-default exit times $\bar{\tau}_i$ for the i^{th} firm are known if the default or other exit occurs after time $t = \Delta t$ and at or before $t = N\Delta t$. The possible values for τ_i and $\bar{\tau}_i$ are integers between 2 and N , inclusive. If a firm exits before the month end, then the exit time is recorded as the first month end after the exit. If the firm does not exit before $t = N\Delta t$, then the convention can be used that both of these values are infinite. If the firm has a default type of exit within the dataset, then $\bar{\tau}_i$ can be considered as infinite. If instead the firm has a non-default type of exit within the dataset, then τ_i can be considered as infinite. The set of all default times and non-default exit times for

all firms is denoted by τ and $\bar{\tau}$, respectively. The first month in which firm i has an observation is denoted by t_{0i} . Except for cases of missing data, these observations continue until the end of the dataset if the firm never exits. If the firm does exit, the last needed input variable $X_i(n)$ is for $n = \min(\tau_i, \bar{\tau}_i) - 1$.

The calibration of the β and $\bar{\beta}$ parameters is done by maximizing a pseudo-likelihood function. The function to be maximized violates the standard assumptions of likelihood functions, but Appendix A in Duan et al. (2011) derives the large sample properties of the pseudo-likelihood function.

In formulating the pseudo-likelihood function, the assumption is made that the firms are conditionally independent from each other. In other words, correlations arise naturally from sharing common factors $W(n)$ and any correlations there are between different firms' firm-specific variables. With this assumption, the pseudo-likelihood function for horizon of l months, a set of parameters β and $\bar{\beta}$ and the dataset $(\tau, \bar{\tau}, X)$ is:

$$\mathcal{L}_l(\beta, \bar{\beta}; \tau, \bar{\tau}, X) = \prod_{m=1}^{N-1} \prod_{l=1}^l P_l(\beta, \bar{\beta}; \tau_i, \bar{\tau}_i, X_i(m)). \quad (10)$$

Here, $P_l(\beta, \bar{\beta}; \tau_i, \bar{\tau}_i, X_i(m))$ is a probability for firm i , with the nature of the probability depending on what happens to the firm during the period from month m to month $m+l$. This is defined as:

$$\begin{aligned}
& P_l(\beta, \bar{\beta}; \tau_i, \bar{\tau}_i, X_i(m)) \\
& = \mathbf{1}_{\{t_{0i} \leq m, \min(\tau_i, \bar{\tau}_i) > m+l\}} \\
& \quad \times \exp \left\{ -\Delta t \sum_{j=0}^{l-1} \left[H(\beta(j), X_i(m)) + H(\bar{\beta}(j), X_i(m)) \right] \right\} \\
& + \mathbf{1}_{\{t_{0i} \leq m, \tau_i \leq \bar{\tau}_i, \tau_i \leq m+l\}} \{ 1 - \exp[-\Delta t H(\beta(\tau_i - m - 1), X_i(m))] \} \\
& \quad \times \exp \left\{ -\Delta t \sum_{j=0}^{\tau_i - m - 2} \left[H(\beta(j), X_i(m)) + H(\bar{\beta}(j), X_i(m)) \right] \right\} \\
& + \mathbf{1}_{\{t_{0i} \leq m, \bar{\tau}_i < \tau_i, \bar{\tau}_i \leq m+l\}} \{ 1 - \exp[-\Delta t H(\bar{\beta}(\bar{\tau}_i - m - 1), X_i(m))] \} \\
& \quad \times \exp[-\Delta t H(\beta(\tau_i - m - 1), X_i(m))] \\
& \quad \times \exp \left\{ -\Delta t \sum_{j=0}^{\bar{\tau}_i - m - 2} \left[H(\beta(j), X_i(m)) + H(\bar{\beta}(j), X_i(m)) \right] \right\} \\
& + \mathbf{1}_{\{t_{0i} > m\}} + \mathbf{1}_{\{\min(\tau_i, \bar{\tau}_i) \leq m\}}. \quad (11)
\end{aligned}$$

In words, if firm i survives from the observation time at month m for the full horizon l until at least $m + l$, then the probability is the model-based survival probability for this period. This is the first term in (11). The second term handles the cases where the firm has a default within the horizon, in which case the probability is the model-based probability of the firm defaulting at the month that it ends up defaulting, as given in equation (8). The third term handles the cases where the firm has a non-default exit within the horizon, in which case the probability is the model-based probability of the firm having a non-default type exit at the month that the exit actually does occur. The expression for this probability uses the conditional non-default type exit probability given in equation (4). The final two terms handle the cases where the firm is not in the data set at month m — either the first observation for the firm is after m or the firm has already exited. A constant value is assigned in this case so that this firm will not affect the maximization at this time point.

The pseudo-likelihood function given in (10) can be numerically maximized to give estimates for the coefficients β and $\bar{\beta}$. Notice though that the sample observations for the pseudo-likelihood function are overlapping if the horizon is longer than one month. For example, when $l = 2$, default over the next two periods from month m is correlated to default over the next two periods from month $m + 1$ due to the common month in the two sample observations. However, in Appendix A of Duan et al. (2011), the maximum pseudo-likelihood estimator is shown to be consistent, in the sense that the estimators converge to the “true” parameter value in the large sample limit.

It would not be feasible to numerically maximize the pseudo-likelihood function using the expression given in (11), due to the large dimension of the β and $\bar{\beta}$ parameters. Notice though that each of the terms in (11) can be written as a product of terms containing only β and terms containing only $\bar{\beta}$. This will allow separate maximizations with respect to β and with respect to $\bar{\beta}$.

The β and $\bar{\beta}$ specific versions of (11) are:

$$\begin{aligned}
& P_i^\beta(\beta; \tau_i, \bar{\tau}_i, X_i(m)) \\
&= 1_{\{t_{0i} \leq m, \min(\tau_i, \bar{\tau}_i) > m+l\}} \exp \left\{ -\Delta t \sum_{j=0}^{l-1} H(\beta(j), X_i(m)) \right\} \\
&+ 1_{\{t_{0i} \leq m, \tau_i \leq \bar{\tau}_i, \tau_i \leq m+l\}} \exp \left\{ -\Delta t \sum_{j=0}^{\tau_i - m - 2} H(\beta(j), X_i(m)) \right\} \\
&\quad \times \{1 - \exp[-\Delta t H(\beta(\tau_i - m - 1), X_i(m))]\} \\
&+ 1_{\{t_{0i} \leq m, \bar{\tau}_i < \tau_i, \bar{\tau}_i \leq m+l\}} \exp \left\{ -\Delta t \sum_{j=0}^{\bar{\tau}_i - m - 2} H(\beta(j), X_i(m)) \right\} \\
&\quad \times \exp[-\Delta t H(\beta(\tau_i - m - 1), X_i(m))] \\
&+ 1_{\{t_{0i} > m\}} + 1_{\{\min(\tau_i, \bar{\tau}_i) \leq m\}}, \\
& P_i^{\bar{\beta}}(\bar{\beta}; \tau_i, \bar{\tau}_i, X_i(m)) \\
&= 1_{\{t_{0i} \leq m, \min(\tau_i, \bar{\tau}_i) > m+l\}} \exp \left\{ -\Delta t \sum_{j=0}^{l-1} H(\bar{\beta}(j), X_i(m)) \right\} \\
&+ 1_{\{t_{0i} \leq m, \tau_i \leq \bar{\tau}_i, \tau_i \leq m+l\}} \exp \left\{ -\Delta t \sum_{j=0}^{\tau_i - m - 2} H(\bar{\beta}(j), X_i(m)) \right\} \\
&+ 1_{\{t_{0i} \leq m, \bar{\tau}_i < \tau_i, \bar{\tau}_i \leq m+l\}} \exp \left\{ -\Delta t \sum_{j=0}^{\bar{\tau}_i - m - 2} H(\bar{\beta}(j), X_i(m)) \right\} \\
&\quad \times \{1 - \exp[-\Delta t H(\bar{\beta}(\tau_i - m - 1), X_i(m))]\} \\
&+ 1_{\{t_{0i} > m\}} + 1_{\{\min(\tau_i, \bar{\tau}_i) \leq m\}}. \tag{12}
\end{aligned}$$

Then, the β and $\bar{\beta}$ specific versions of the pseudo-likelihood function are given by:

$$\begin{aligned}
\mathcal{L}_l^\beta(\beta; \tau, \bar{\tau}, X) &= \prod_{m=1}^{N-1} \prod_{i=1}^I P_i^\beta(\beta; \tau_i, \bar{\tau}_i, X_i(m)), \tag{13} \\
\mathcal{L}_l^{\bar{\beta}}(\bar{\beta}; \tau, \bar{\tau}, X) &= \prod_{m=1}^{N-1} \prod_{i=1}^I P_i^{\bar{\beta}}(\bar{\beta}; \tau_i, \bar{\tau}_i, X_i(m)).
\end{aligned}$$

With the definitions given in (12) and (13), it can be seen that:

$$\mathcal{L}_l(\beta, \bar{\beta}; \tau, \bar{\tau}, X) = \mathcal{L}_l^\beta(\beta; \tau, \bar{\tau}, X) \mathcal{L}_l^{\bar{\beta}}(\bar{\beta}; \tau, \bar{\tau}, X). \tag{14}$$

Thus, \mathcal{L}_l^β and $\mathcal{L}_l^{\bar{\beta}}$ can be separately maximized to find their respective parameters. A further important separation is a separation by horizons. Notice that we can decompose P_i^β and $P_i^{\bar{\beta}}$ as:

$$P_i^{\beta}(\beta; \tau_i, \bar{\tau}_i, X_i(m)) = \prod_{l'=0}^{l-1} P^{\beta(l')}(\beta(l'); \tau_i, \bar{\tau}_i, X_i(m)), \quad (15)$$

$$P_i^{\bar{\beta}}(\bar{\beta}; \tau_i, \bar{\tau}_i, X_i(m)) = \prod_{l'=0}^{l-1} P^{\bar{\beta}(l')}(\bar{\beta}(l'); \tau_i, \bar{\tau}_i, X_i(m)),$$

where:

$$\begin{aligned} & P^{\beta(l')}(\beta(l'); \tau_i, \bar{\tau}_i, X_i(m)) \\ &= 1_{\{t_{0i} \leq m, \min(\tau_i, \bar{\tau}_i) > m+l'+1\}} \exp[-\Delta t H(\beta(l'), X_i(m))] \\ &+ 1_{\{t_{0i} \leq m, \tau_i \leq \bar{\tau}_i, \tau_i = m+l'+1\}} \{1 - \exp[-\Delta t H(\beta(l'), X_i(m))]\} \\ &+ 1_{\{t_{0i} \leq m, \bar{\tau}_i < \tau_i, \bar{\tau}_i = m+l'+1\}} \exp[-\Delta t H(\beta(l'), X_i(m))] \\ &+ 1_{\{t_{0i} > m\}} + 1_{\{\min(\tau_i, \bar{\tau}_i) < m+l'+1\}}, \\ & P^{\bar{\beta}(l')}(\bar{\beta}(l'); \tau_i, \bar{\tau}_i, X_i(m)) \\ &= 1_{\{t_{0i} \leq m, \min(\tau_i, \bar{\tau}_i) > m+l'+1\}} \exp[-\Delta t H(\bar{\beta}(l'), X_i(m))] \\ &+ 1_{\{t_{0i} \leq m, \tau_i \leq \bar{\tau}_i, \tau_i = m+l'+1\}} \\ &+ 1_{\{t_{0i} \leq m, \bar{\tau}_i < \tau_i, \bar{\tau}_i = m+l'+1\}} \{1 - \exp[-\Delta t H(\bar{\beta}(l'), X_i(m))]\} \\ &+ 1_{\{t_{0i} > m\}} + 1_{\{\min(\tau_i, \bar{\tau}_i) < m+l'+1\}}. \end{aligned} \quad (16)$$

Thus, the β and $\bar{\beta}$ specific pseudo-likelihood functions can be decomposed as:

$$\begin{aligned} \mathcal{L}_i^{\beta}(\beta; \tau, \bar{\tau}, X) &= \prod_{l'=0}^{l-1} \mathcal{L}^{\beta(l')}(\beta(l'); \tau, \bar{\tau}, X) \\ \mathcal{L}_i^{\bar{\beta}}(\bar{\beta}; \tau, \bar{\tau}, X) &= \prod_{l'=0}^{l-1} \mathcal{L}^{\bar{\beta}(l')}(\bar{\beta}(l'); \tau, \bar{\tau}, X) \end{aligned} \quad (17)$$

where,

$$\begin{aligned} \mathcal{L}^{\beta(l')}(\beta(l'); \tau, \bar{\tau}, X) &= \prod_{m=1}^{N-1} \prod_{i=1}^I P^{\beta(l')}(\beta(l'); \tau_i, \bar{\tau}_i, X_i(m)) \\ \mathcal{L}^{\bar{\beta}(l')}(\bar{\beta}(l'); \tau, \bar{\tau}, X) &= \prod_{m=1}^{N-1} \prod_{i=1}^I P^{\bar{\beta}(l')}(\bar{\beta}(l'); \tau_i, \bar{\tau}_i, X_i(m)) \end{aligned} \quad (18)$$

Thus, for every horizon l' , $\mathcal{L}^{\beta(l')}(\beta(l'); \tau, \bar{\tau}, X)$ and $\mathcal{L}^{\bar{\beta}(l')}(\bar{\beta}(l'); \tau, \bar{\tau}, X)$ can be separately maximized. In summary, for the current CRI implementation where the horizons are from one month to 24 months, and where there are 13 variables, a $2 \times 24 \times 13$ dimensional maximization is turned into a 13 dimensional maximization done 2×24 times. This makes the

calibration problem tractable. Additional implementation details on the calibration are given in Section 3.

II. INPUT VARIABLES AND DATA

Subsection 2.1 describes the input variables used in the quantitative model. Currently, the same description of input variables is common to all the economies under the CRI's coverage. Future enhancements to the CRI system will allow different input variables for different economies. The effect of each of the variables on the PD output will be discussed in the empirical analysis of Section 4.

Subsection 2.2 gives the data sources and relevant details of the data sources. There are two categories of data sources: current and historical. Data sources used for current data need to be updated in a timely manner so that daily updates of PDs are meaningful. They also need to be comprehensive in their current coverage of firms. Data sources that are comprehensive for current data may not necessarily have comprehensive historical coverage for different economies. Other data sources are thus merged in order to obtain comprehensive coverage for historical and current data.

Subsection 2.3 indicates the fields from the data sources that are used to construct the input variables. For some of the fields, proxies need to be used for a firm if the preferred field is not available for that firm.

Subsection 2.4 discusses the definition and sources of defaults and of other exits used in the CRI.

2.1 Input Variables

Following the notation that was introduced in Section 2, firm i 's input variables at time $t = n\Delta t$ are represented by the vector $X_i(n) = (W(n), U_i(n))$ consisting of a vector $W(n)$ that is common to all firms in the same economy, and a firm-specific vector $U_i(n)$ which is observable from the date the firm's first financial statement is released, until the month end before the month in which the firm exits, if it does exit.

In Duan et al. (2011), different variables that are commonly used in the literature were tested as candidates for the elements of $W(n)$ and $U_i(n)$. Two

common variables and ten firm-specific variables, as described below, were selected as having the greatest predictive power for corporate defaults in the United States. In the current stage of development, this same set of twelve input variables is used for all economies. Future development will include variable selection for firms in different economies.

Common variables

The vector $W(n)$ contains two elements, consisting of:

1. Stock index return: the trailing one-year simple return on a major stock index of the economy.
2. Interest rate: a representative 3-month short term interest rate.

Firm-specific variables

The ten firm-specific input variables are transformations of measures of six different firm characteristics. The six firm characteristics are: (i) volatility-adjusted leverage; (ii) liquidity; (iii) profitability; (iv) relative size; (v) market mis-valuation/future growth opportunities; and (vi) idiosyncratic volatility.

Volatility-adjusted leverage is measured as the distance-to-default (DTD) in a Merton-type model.

The calculation of DTD used by the CRI allows a meaningful DTD for financial firms, a critical sector that must be excluded from most DTD computations. This calculation is detailed in Section 3.

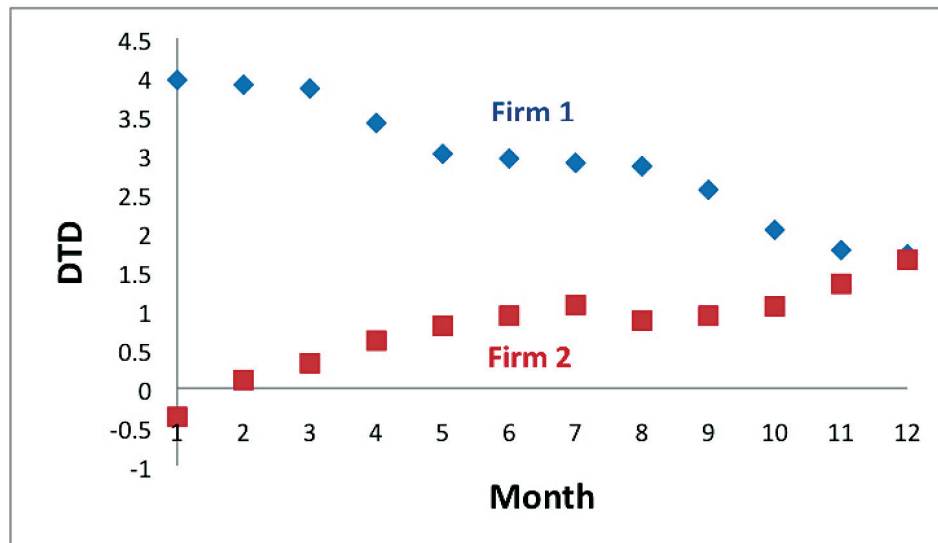
Liquidity is measured as a ratio of cash and short term investments to total assets, profitability is measured as a ratio of net income to total assets, and relative size is measured as the logarithm of the ratio of market capitalization to the economy's median market capitalization.

Duan et al. (2011) transformed these first four characteristics into level and trend versions of the measures. For each of these, the level is computed as the one-year average of the measure, and the trend is computed as the current value of the measure minus the one-year average of the measure. The level and trend of a measure has seldom been used in the academic or industry literature for default prediction, and Duan et al. (2011) found that using the level and trend significantly improves the predictive power of the model for short-term horizons.

To understand the intuition behind using level and trend of a measure as opposed to using just the current value, consider the case of two firms with the same current value for all measures. If the level and trend transformations were not performed, then only the

FIGURE 2

Two firms with all current values equal to each other, but DTD trending in the opposite direction.



current values would be used and the two firms would have identical PD. Suppose that for the first firm the DTD had reached its current level from a high level, and for the second firm the DTD had reached its current level from a lower level (see Figure 2). The first firm's leverage is increasing (worsening) and the second firm's leverage is decreasing (improving). If there is a momentum effect in DTD, then firm 1 should have a higher PD than firm 2.

Duan et al. (2011) found evidence of the momentum effect in DTD, liquidity, profitability and size. For the other two firm characteristics, applying the level and trend transformation did not improve the predictive power of the model.

One of the remaining two firm characteristics is the market mis-valuation/future growth opportunities characteristic, which is taken as the market-to-book asset ratio and measured as a ratio of market capitalization and total liabilities to total assets. One can see whether the market mis-valuation effect or the future growth opportunities effect dominates this measure by looking at whether the parameter for this variable is positive or negative. This will be further discussed in the empirical analysis of Section 4.

The final firm characteristic is the idiosyncratic volatility which is taken as sigma, following Shumway (2001). Sigma is computed by regressing the monthly returns of the firm's market capitalization on the monthly returns of the economy's stock index, for the previous 12 months. Sigma is defined to be the standard deviation of the residuals of this regression. Shumway (2001) reasons that sigma should be logically related to bankruptcy since firms with more variable cash flows and therefore more variable stock returns relative to a market index are likely to have a higher probability of bankruptcy.

Finally, the vector $U_i(n)$ contains ten elements, consisting of:

1. Level of DTD.
2. Trend of DTD.
3. Level of (Cash + Short term investments)/Total assets, abbreviated as CASH/TA.

4. Trend of CASH/TA.
5. Level of Net income/Total Assets, abbreviated as NI/TA.
6. Trend of NI/TA.
7. Level of log (Firm market capitalization/Economy's median market capitalization), abbreviated as SIZE.
8. Trend of SIZE.
9. Current value of (Market capitalization + total liabilities)/Total asset, abbreviated as M/B.
10. Current value of SIGMA.

The data fields that are needed to compute DTD and Short term investments are described in Subsection 2.3. The remaining data fields required are straightforward and standard. The computation for DTD is explained in Section 3.

2.2 Data Sources

There are two data sources that are used for the daily PD updates: Thomson Reuters Datastream and the Bloomberg Data License Back Office Product. Many of the common factors such as stock index prices and short term interest rates are retrieved from Datastream.

Firm-specific data comes from Bloomberg's Back Office Product which delivers daily update files by region via FTP after the respective market closes. All relevant data is extracted from the FTP files and uploaded into the CRI database for storage. From this, the necessary fields are extracted and joined with previous months of data.

The Back Office Product includes daily market capitalization data based on closing share prices and also includes new financial statements as companies release them. Firms will often have multiple versions of financial statements within the same period, with different accounting standards, filing statuses (most recent, preliminary, original, reclassified or restated), currencies or consolidated/unconsolidated indicators. A major challenge lies in prioritizing these financial

statements to decide which data should be used. The priority rules are described in Section 3.

The firm coverage of the Back Office Product is of sufficient quality that nearly 30,000 firms can be updated on a daily basis in the 30 economies under the CRI's coverage. While the current coverage is quite comprehensive, historical data from the Back Office Product can be sparse for certain economies. For this reason, various other databases are merged in order to fill out the historical data. The other databases used for historical data are: a database from the Taiwan Economics Journal (TEJ) for Taiwanese firms, a database provided by Korea University for South Korean firms, and data from Prowess for Indian firms.

With all of the databases merged together and for the 30 economies under CRI's coverage, over 50,000 exchange listed firms are in the CRI database. This includes over 20,000 delisted firms. The historical coverage of the firm data goes back to the early 1990's.

2.3 Constructing Input Variables

The chosen stock indices and short term interest rates for the 30 economies under the CRI's current coverage are listed in Table A.2 and Table A.3, respectively. All economies are listed by their three letter ISO code given in Table A.1.

Most of the firm-specific variables can be readily constructed from standard fields from firms' financial statements in addition to daily market capitalization values. The only two exceptions are the DTD and the liquidity measure.

The calculation for DTD is explained in Section 3. In the calculation, several variables are required. One variable is a proxy for a one-year risk-free interest rate, and the choices for each of the 30 economies are listed in Table A.4. Total assets, long term borrowing and total liabilities are also required, but are standard financial statement fields and present no difficulties.

Total current liabilities are also required, and due to the relatively large numbers of firms that are missing

this value, proxies had to be found. The preferred Bloomberg field for this is BS_CUR_LIAB. If this is missing, then the sum of BS_ST_BORROW, BS_OTHER_ST_LIAB and BS_CUST_ACCPT_LIAB_CUSTDY_SEC (customers' acceptance and liabilities/custody securities) is used. If one or two of these are missing, zero is inserted for those fields, but at least one field is required.

The liquidity measure requires different fields between financial and non-financial firms. For non-financial firms, the numerator (Cash+Short-term investments) of the ratio is taken as the sum of BS_CASH_NEAR_CASH_ITEM and BS_MKT_SEC_OTHER_ST_INVEST (marketable securities and other short term investments). If BS_MKT_SEC_OTHER_ST_INVEST is missing, replace it with zero (but BS_CASH_NEAR_CASH_ITEM is required).

It was found that this sum frequently overstated the liquidity for financial firms. In place of BS_MKT_SEC_OTHER_ST_INVEST, financial firms use the sum of ARD_SEC_PURC_UNDER_AGR_TO_RESELL (securities purchased under agreement to re-sell), ARD_ST_INVEST and BS_INTERBANK_ASSET. If one or two of these are missing, zero is inserted for those fields, but at least one field is required. The "ARD_" prefix indicates that these are "as reported" numbers directly from the financial statements. As such, for some firms these fields may need to be adjusted to the same units before adding them to other fields.

Table A.5 contains summary statistics of the firm-specific variables: DTD, CASH/TA, NI/TA, SIZE, M/B, and SIGMA, with the summary statistics provided for firms grouped by economy.

2.4 Data for Defaults

The CRI database contains credit events of over 4,000 firms from 1990 to the present. The defaults events come from numerous sources, including Bloomberg, Compustat, CRSP, Moody's reports, TEJ, exchange web sites and news sources.

The CRI system considers two broad categories of default: bankruptcy filings and default corporate actions. Within bankruptcies, the sub-categories are listed in Table A.6. Delistings that are labeled as being due to bankruptcy are categorized as a bankruptcy filing and the delisting date is used if the actual bankruptcy filing event date cannot be found.

Default corporate actions can include missed or delayed interest or principal payments by the due date, or as debt restructuring. The more precise sub-categories of default corporate actions used by the CRI are also listed in Table A.6.

The exit events that are not considered as defaults in the CRI system are listed in Table A.7. Firms that are delisted from an exchange and then experience a default event within 365 calendar days of the delisting have the exit event re-classified as a credit default. This is consistent with Shumway (2001) who re-classifies a delisting event to credit default if there is any bankruptcy event within five years after the delisting event.

In addition to the aforementioned events, there are various cases that require special attention. A non-exhaustive overview of such events and the way they are treated in the CRI system is listed below. The treatments of these events are compared to the treatments by the three major credit rating agencies (CRAs), as described in Moody's Investor Services (2011), Standard & Poors (2011) and Fitch Ratings (2011).

- **Missed or delayed payments made within the grace period are not counted as defaults:** The major CRAs forgive such events as they focus on the ability or willingness to pay when assessing credit risks of an issuer. Therefore, a payment made within the grace period is seen as the issuer being willing to uphold the debt contract. Considering that the typical grace period for missed interest or principal payment is 7 to 14 days, this criterion is within the Basel II guidelines (Basel Committee on Banking Supervision, 2006), which allows a 90 day grace period.

- **Related obligor default will be assessed on a case-by-case basis:** The CRI, like major CRAs, does not consider related party-default (e.g. subsidiary bankruptcy) as a default event. However, the Monetary Authority of Singapore (2004) advises that related obligor defaults can be default events depending on the economic dependence and integration between the subsidiary and the parent company. When a non-operating holding parent company relies heavily on its subsidiary, bankruptcy by the subsidiary will cause a considerable economic impact on the parent company. These cases will be carefully reviewed.
- **Selective default on one obligation but not on the others is counted as a default event:** The CRI considers default on one of the obligations but not the others as default by the overall company. This is because, in general, a default in one obligation is a sign of business deterioration and often is followed by a bankruptcy filing. S&P and Moody's consider selective default as default. Fitch Ratings categorizes such events as a "Restricted Default".

Another important challenge in identifying default events is linked to the fact that definitions of credit default can vary across different jurisdictions and between different data sources. An area of continuing development is in normalizing to a common set of definitions.

Table A.8 lists the total number of firms, number of defaults and number of other exits in each of the 30 economies each year from 1992 to 2011. Note that the total number of firms here includes all firms where the *primary listing of the shares are on an exchange in that economy* and may include firms where there are too many missing data values for a PD estimate to be made. However, the number of firms listed on the CRI web portal under the tab "Aggregate Forecast" includes firms that are *domiciled in that economy* and excludes firms where a PD cannot be produced due to missing data.

III. IMPLEMENTATION DETAILS

Section 1 described the modeling framework underlying the current implementation of the CRI rating system. It focused on theory rather than the details encountered in an operational implementation. The present section describes how the CRI system handles these more specific issues.

Subsection 3.1 describes implementation details related to data, mainly dealing with data cleaning and missing data. Subsection 3.2 describes the specific computation of distance-to-default (DTD) used by the CRI system that leads to meaningful DTD for financial firms. Subsection 3.3 explains how the calibration previously described in Subsection 1.2 can be implemented. Subsection 3.4 gives the implementation details relevant to the daily output. This includes an explanation of the various modifications needed to compute daily PD so that the daily PD is consistent with the usual month end PD, and a description of the computation of the aggregate PDs provided by the CRI.

3.1 Data Treatment

Fitting data to monthly frequency: Historical end of month data for every firm in an economy is required to calibrate the model. For daily data such as market capitalization, interest rates and stock index values, the last day of the month for which there is valid data is used.

For financial statement variables, data is used starting from the period end of the statement lagged by three months. This is to ensure (insofar as is possible) that predictions are made based on information that was available at the time the prediction was made. Of course, for more recent data where the CRI database contains the financial statement but the period end lagged by three months is after the current day, the financial statement is used in computing PD. The CRI considers financial statement variables to be valid for one year without restriction after they are first used.

Currency conversions are required if the market capitalization or any of the financial statement variables are reported in a currency different than the currency

of the economy. If a currency conversion is required, the foreign exchange rate used is that reported at the relevant market close. For firms traded in Asia and Asia-Pacific, the Tokyo closing rate is used; for firms traded in Western Europe, the London closing rate is used; and for firms traded in North America, the New York closing rate is used. For market capitalizations, the FX rate used is for the date that the market capitalization is reported. For financial statement variables, the FX rate used is for the date of the period end of the statement.

Priority of financial statements: As described in Subsection 2.2, data provided in Bloomberg's Back Office Product can include numerous versions of financial statements within the same period. If there are multiple financial statements with the same period end, priority rules must be followed in order to determine which to use. The formulation and implementation of these rules is a major challenge and an area of continuing development. The current priority rules are as follows.

The first rule prioritizes by consolidated/unconsolidated status. This status is relevant only to firms in India, Japan, South Korea and Taiwan, so this rule is only relevant in those economies. Most firms in these economies issue unconsolidated financial statements more frequently than consolidated ones, so these are given higher priority. This simple prioritization can, however, lead to cases where the financial statements used switch from consolidated statements to unconsolidated statements and back again. A more complex prioritization rule is currently under development, with the intention of avoiding this situation.

If, after the first prioritization rule has been applied, there are still multiple financial statements, the second rule is applied. This is prioritization by fiscal period. In most economies, annual statements are required to be audited, whereas other fiscal periods are not necessarily audited. The order of priority from highest to lowest is, therefore: annual, semi-annual, quarterly, cumulative, and finally other fiscal periods. The one variable that is currently an exception to this rule is net income.

Net income is a flow variable so adjustments need to be made to annualize the net income from non-annual financial statements. Study needs to be done to determine whether seasonal effects will unduly affect PD estimates before net income from non-annual statements can be introduced.

The third prioritization rule is based on filing status. The “Most Recent” statement is used before the “Original” statement, which is used before the “Preliminary” statement.

The final prioritization rule is based on the accounting standard. Here, financial statements that are reported using Generally Accepted Accounting Principles (GAAP) are given higher priority than financial statements that are reported using International Financial Reporting Standards (IFRS). If an accounting standard is not indicated at all, the financial statement is not used.

Provisions for missing values and outliers: Missing values and outliers are dealt with by a three step procedure. In the first step, the ten firm-specific input variables are computed for all firms and all months. In the second step, outliers are eliminated by winsorization. In the final step, missing values are replaced under certain conditions.

The first step is to compute the input variables and determine which are missing. As mentioned previously, financial statement variables are carried forward for one year after the date that they are first used. This is generally three months after the period end of the statement. If no financial statement is available for the company within this year, then the financial statement variable will be missing. For market capitalization, if there is no valid market capitalization value within the calendar month, then the value is set to missing.

For illiquid stocks, if there has been no valid market capitalization value for a firm within the last 90 calendar days, then the market capitalization is deemed to not properly reflect the value of the firm. The firm is considered to have exited with a non-default event. Once the firm starts trading again and a new financial statement is released, the firm can enter back into the

calibration. With regard to historical PD, the PD can be reported again once there are enough valid variables.

With regard to the level variables, the current month and the last eleven months are averaged to compute the level. There is no lower limit on the number of valid observations. Only if all of the values are missing is the level variable considered to be missing.

For the trend variable, the level is subtracted from the current month. If the current month is missing, then the trend variable is set to missing.

The value of M/B is set to missing if any of the following values: market capitalization, total liabilities or total assets of the firm, are missing. For the computation of SIGMA, seven valid returns over the last twelve months of possible returns are required for the regression. If there are less than seven valid returns, SIGMA is set to missing.

In this way, the eight trend and level variables plus M/B and SIGMA are computed and evaluated as missing or present. Winsorization can then be performed as a second step to eliminate outliers. The volume of outliers is too large to be able to determine whether each one is valid or not, so winsorization applies a floor and a cap on each of the variables. The historical 0.1 percentile and 99.9 percentile for all firms in the economy are recorded for each of the ten variables. Any values that exceed these levels are set to equal these boundary values.

With a winsorization level and 0.1 percentile and 99.9 percentile, the boundary values still may not be reasonable. For example, NI/TA levels of nearly -25 have been observed at this stage. In these cases, a more aggressive winsorization level is applied, until the boundary values are reasonable. Thus, the winsorization level is economy and variable specific, and will depend on the data quality for that economy and variable. Winsorization levels different than the default of 0.1 percentile and 99.9 percentile are indicated in Table A.5.

A third and final step can be taken to deal with missing values. If, during a particular month, no variables for a firm are missing, then the PD can be computed. If six or more of these ten variables are missing, there

are deemed to be too many missing observations and no replacements are made.

If between one and five variables are missing out of the ten, the first step is to trace back for at most twelve months to use previous values of these variables instead. If this does not succeed in replacing all of the variables, a replacement by sector medians is done. The median is for the financial or non-financial firms (as indicated by their Level I Bloomberg Industry Classification System) within the economy during that month. Replacement by the sector median should have a neutral effect on the PD of the firm; the firm is assessed by the other variables that it does have values for. This sector median is always performed in calibration. However, when reporting historical PD, the sector replacement is not done if it results in a relative change in PD of 10% or more where the initial PD was at or above 100bps, or an absolute change in PD of 10bps or more where the initial PD was below 100bps.

Inclusion/exclusion of companies for calibration:

Firms are included within an economy for calibration when the primary listing of the firm is on an exchange in the economy. This ensures that all firms within the economy are subject to the same disclosure and accounting rules.

There are a relatively small number of firms that are dual listed, in which two corporations listed in different exchanges operate as a single entity but retain separate legal status. In the CRI system, a combined company will be assigned to the single economy it is most associated with. An example is the Rio Tinto Group. This consists of Rio Tinto plc, listed in the UK; and Rio Tinto Limited, listed in Australia. Most of Rio Tinto's operations are in Australia rather than the UK, so Rio Tinto is assigned to Australia.

In the US, firms traded on the OTC markets or the Pink Sheets are not considered as exchange listed so are not included in calibration or in the reporting of PDs. Many of these firms are small or start-up firms. Including this large group of companies would skew the calibration and the aggregate results. The TSX Venture Exchange in Canada also contains only small and

start-up firms, so firms listed here are also excluded.

Other examples include Taiwan's GreTai Securities Market and Singapore's Catalist. The challenge for markets outside of the US or Canada is that the data on whether firms are listed on the smaller markets rather than the main board is difficult to obtain. For all economies besides the US and Canada, there is continuing work being done in the CRI system to exclude firms that are not listed on major exchanges within a country.

Firms that record an exit (other than due to no trading for 90 calendar days) are not entered back into the calibration even if the firm continues to trade and issue financial statements, as can happen after firms declare bankruptcy. There are two exceptions to this exclusion. The first, determined on a case by case basis, is if the firm should be deemed to have re-emerged from bankruptcy. The second exception is for all firms in China, where two situations are prevalent. The first situation is that the firm experiences few repercussions from the default and continues operating normally. The other situation is for one firm to take over a defaulted firm's listing. This happens due to the limited supply of exchange listings. Because of these two situations, the norm for firms based in China is to emerge from a default, so the CRI system enters all of these companies back into the calibration as new companies.

3.2 Distance-to-Default Computation

The distance-to-default (DTD) computation used in the CRI system is not a standard one. Standard computations exclude financial firms, but excluding the financial sector means neglecting a critical part of any economy. So the standard DTD computation must be extended to give meaningful estimates for financial firms as well. The description of the specialized DTD computation starts with a brief description of the Merton (1974) model. Merton's model makes the simplifying assumption that firms are financed by equity and a single zero-coupon bond with maturity date T and principal L . The asset value of the firm V_t follows a geometric Brownian motion:

$$dV_t = \mu V_t dt + \sigma V_t dB_t. \quad (19)$$

Here, B_t is standard Brownian motion, μ is the drift of the asset value in the physical measure and σ is the volatility of the asset value. Equity holders receive the excess value of the firm above the principal of the zero-coupon bond and have limited liability, so the equity value at maturity is: $E_T = \max(V_T - L, 0)$. This is just a call option payoff on the asset value with a strike value of L . Thus, the Black-Scholes option pricing formula can be used for the equity value at times t before T :

$$E_t = V_t N(d_1) - e^{-r(T-t)} L N(d_2), \quad (20)$$

where r is the risk-free rate, $N(\cdot)$ is the standard normal cumulative distribution function, and:

$$d_{1,2} = \frac{\log\left(\frac{V_t}{L}\right) + \left(r \pm \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}. \quad (21)$$

In Merton's model, DTD is defined as volatility scaled distance of the expected asset value under the physical measure at maturity T from the default point L :

$$\text{DTD}_t = \frac{\log\left(\frac{V_t}{L}\right) + \left(\mu - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}. \quad (22)$$

The standard KMV assumptions given in Crosbie and Bohn (2003) are to set the time to maturity $T-t$ at a value of one year, and the principal of the zero-coupon bond L to a value equal to the firm's current liabilities plus one half of its long-term debt. Here, the current liabilities and long-term debt are taken from the firm's financial statements. If the firm is missing the current liabilities field, then various substitutes for this field can be used, as described in Subsection 2.3.

This is a poor assumption of the debt level for financial firms, since they typically have large liabilities, such as deposit accounts, that are neither classified as current liabilities nor long-term debt. Thus, using these standard assumptions means ignoring a large part of the debt of financial firms.

To properly account for the debt of financial firms, Duan (2010) includes a fraction δ of a firm's other liabilities. The other liabilities are defined as the firm's total liabilities minus both the short and long-term debt. The debt level L then becomes the current liabilities plus half of the long-term debt plus the fraction δ multiplied by the other liabilities, so that the debt level is a function of δ . The standard KMV assumptions are then a special case where $\delta = 0$.

The fraction δ can be optimized along with μ and σ in the maximum likelihood estimation method developed in Duan (1994, 2000). Following Duan et al. (2011), the firm's market value of assets is standardized by its book value A_t so that the scaling effect from a major investment or financing by the firm will not distort the time series from which the parameter values are estimated. Thus, the log-likelihood function is:

$$\begin{aligned} \mathcal{L}(\mu, \sigma, \delta) &= -\frac{n-1}{2} \log(2\pi) - \frac{1}{2} \sum_{t=2}^n \log(\sigma^2 h_t) - \sum_{t=2}^n \log\left(\frac{\hat{V}_t(\sigma, \delta)}{A_t}\right) \\ &\quad - \sum_{t=2}^n \log\left[N\left(\hat{d}_1(\hat{V}_t(\sigma, \delta), \sigma, \delta)\right)\right] \\ &\quad - \sum_{t=2}^n \frac{1}{2\sigma^2 h_t} \left[\log\left(\frac{\hat{V}_t(\sigma, \delta)}{A_t} \times \frac{A_{t-1}}{\hat{V}_{t-1}(\sigma, \delta)}\right) - \left(\mu - \frac{\sigma^2}{2}\right) h_t \right]. \end{aligned} \quad (23)$$

Here, n is the number of days with observations of the equity value in the sample, \hat{V}_t is the implied asset value found by solving equation (20), \hat{d}_1 is computed with equation (21) using the implied asset value, and h_t is the number of trading days as a fraction of the year between observations $t-1$ and t . Notice that the implied asset value and \hat{d}_1 are dependent on δ by virtue of the dependence of L on δ .

Implementation of DTD computation: The DTD at the end of each month is needed for every firm in order to calibrate the forward intensity model. A moving window, consisting of the last one year of data before each month end is used to compute the month end DTD. Daily market capitalization data based on closing prices is used for the equity value in the implied asset value computation of equation (20). If there are fewer than 50 days of valid observations for the market

capitalization, then the DTD value is set to missing. An observation is valid if there is positive trading volume that day. If the trading volume is not available, the observation is assumed to be valid if the value for the market capitalization changes often enough. The precise criterion is as follows: if the market capitalization does not change for three days or more in a row, the first day is taken as a valid observation and the remaining days with the same value are set to missing.

The log-likelihood function given in (23) can be maximized as a three dimensional maximization problem over μ , σ and δ . After estimates for these three variables are made, the DTD can be computed from equation (22).

However, with quarterly financial statements there will never be more than three changes in the corporate structure (defined in this model by L and A_t) throughout the year, leading to possibly unstable estimates of δ . This problem is mitigated by performing a two stage optimization for μ , σ and δ .

In the first stage, the optimization for each firm is performed over all three variables. For each firm, in the first month in which DTD can be computed the optimization is unconstrained in μ and σ , while δ is constrained to being in the unit interval $[0,1]$. Thereafter, at month n , the optimization is still unconstrained in μ and σ while δ is constrained to the interval $[\max(0, \hat{\delta}_{n-1} - 0.05), \min(1, \hat{\delta}_{n-1} + 0.05)]$, where $\hat{\delta}_{n-1}$ is the estimate of δ made in the previous month. In other words, a ten percent band around the previous estimate of δ (where that band is floored with 0 and capped with 1) is applied so that the estimates do not fluctuate too much from month to month.

It was found that this was not enough to obtain stable estimates of δ . For many firms, the estimate of δ would frequently lie on the boundary of the constraining interval. To impose greater stability, a second stage is added. At each month end, the average estimate for δ in all financial sector firms in the economy is used for every financial sector firm in the economy, meaning the optimization is only over μ and σ . The same is done for non-financial firms. In fact, the optimization can be reduced to only be over σ by using the sample mean of the log returns of the implied asset values in place of μ .

Since the first stage is done to obtain a stable, sector average estimate of δ , the criteria used to include a firm-month is more strict. In the first stage, a two year window is used instead of one year, and a minimum of 250 days of valid observations of the market capitalization are required instead of 50. If a firm has less than 250 days of valid observations within the last two years of a particular month end, δ will not be estimated for that firm and that month end.

In summary, the DTD for each firm is computed using the economy and sector (financial or non-financial) average for δ in that month, and the estimate of μ and σ based on the last year of data for the firm.

Carrying out this two stage procedure would take several months of computation time on a single PC, given the millions of firm months that are required. However, each of the stages is parallelizable. In the first stage the DTD can be computed independently between firms. In the second stage, once the sector averages of the δ have been computed for each month, the DTD can again be computed independently between firms. In the CRI system, a grid of several hundred computers administered by the NUS Computer Center is used. With this, the DTD computation can be performed for all firms over the full history of twenty years in less than two days.

3.3 Calibration

Implementation: As shown in Section 1, the calibration of the forward intensity model involves multiple maximum pseudo-likelihood estimations, where the pseudo-likelihood functions are given in equation (18). The maximizations are of the logarithm of these expressions, and are performed independently between the default parameters and the exit parameters, and between parameters for different horizons. In the notation of Section 2, the vectors of parameters $\beta(0), \dots, \beta(23)$ and $\bar{\beta}(0), \dots, \bar{\beta}(23)$ are independently estimated.

A few input variables have an unambiguous effect on a firm's probability of default. Increasing values of both the level and trend of DTD, CASH/TA, and

NI/TA all indicate that a firm is becoming more credit worthy and should lead to a decreased PD. For large and relatively clean datasets such as the US, an unconstrained optimization leads to parameter values which largely have the expected sign. For each of DTD level and trend, CASH/TA level and trend, and NI/TA level, the default parameters at all horizons are negative. A negative default parameter at a horizon means that if the variable increases, the forward intensity will decrease (by equation (6)), so that the conditional default probability at that horizon will decrease. The one exception is the NI/TA trend variable. Since the current implementation in the CRI system does not include net income values from quarterly statements, the trending effect is weak.

For some of the smaller economies and economies with lower quality datasets, an unconstrained optimization leads to the default parameters for some of these variables to be positive at several horizons. This leads to counter-intuitive results. For example, if the default parameters for CASH/TA are positive, a firm that increases its cash reserves, all other factors being equal, will have a PD that increases. To prevent such situations, the CRI system performs a constrained optimization with only non-positive values allowed for the default parameters associated with the level and trend of DTD, CASH/TA, and NI/TA.

For this, the Matlab® function “fmincon” from the Optimization Toolbox is used. The analytic gradient and Hessian are not supplied and the algorithm used by “fmincon” is the active-set optimization. If “fmincon” fails to converge, “fminsearch” is used. This uses a simplex search method which takes more time but is generally more likely to converge.

Each evaluation of the pseudo-log-likelihood function can be done in a fraction of second on a standard CPU, even for the largest economies. But since the optimization is over 13 dimensions, thousands of evaluations are required. It is therefore important to make each function evaluation as fast as possible.

Notice that at each time point and at any horizon, there are far more surviving firms than exiting firms. Thus, from equation (16) and (18), it can be seen that the most time-consuming part of evaluating the

pseudo-log-likelihood function is the term for the surviving firms. Evaluating the forward intensity function of equation (7) can be formulated as a matrix-vector multiplication, where the rows of the matrix are the different surviving firms’ variables, and the vector is the vector of parameters. The matrix will typically have several hundreds of thousands of rows and does not change during the optimization (though it will change for different optimizations at different horizons). This type of problem is well-suited for a programmable graphics processing unit (GPU). The CRI system runs the calibrations on an NVIDIA® Tesla C2050 card. For each economy, the calibrations for the default and other exit parameters for horizons up to 24 months typically require ten minutes or less.

Grouping for small economies: There are not enough defaults in some small economies and calibrations of these individual economies are not statistically meaningful. In order to ensure that there are enough defaults for calibration, the 30 economies are categorized into groups according to similarities in their stage of development and their geographic locations. Within these groups the economies are combined and calibrated together. The calibration group for each economy is listed in Table A.1.

Once the economies are combined into groups, the minimum amount of alteration to the input variables is done. Firms will still use the economy-specific variables for the stock index return and short-term interest rate as indicated in Tables A.2 and A.3, and the economy-specific interest rate for the DTD computation as indicated in Table A.4. For the second stage of the DTD computation, described in Subsection 3.2, the sector average estimates for δ are still economy-specific rather than common to the group. For the SIZE variable, the median market capitalization is the economy-specific median. For replacement of missing data, the sector medians are those for the economy. The only exception is for winsorization. So that outliers can be treated consistently, the 0.1 and 99.9 percentile levels (or more aggressive levels) are determined based on all economies within the group.

If the parameters are restricted to being equal in value between all economies in a group, the predictive power of the model can be relatively poor for individual economies. It was found that some of the parameter estimates gave statistically significant differences between economies if the values were allowed to differ. Different estimated parameter values indicate the difference the effect of a variable has on different economies. For example, on estimation it is found that the short-term interest rate has little significance in default prediction for Indonesian firms at short time horizons, but a significant effect on the rest of the Original ASEAN ex-SG group consisting of Malaysia, Philippines and Thailand.

The reason for combining economies into groups is so that the aggregate default experience can be used to estimate the parameters. Different economies are therefore allowed to have different parameter values for a variable only on a selective basis. There are a large number of combinations of sub-groups and variables, and extensive, on-going experiments are being performed on different combinations. Each sub-group must contain at least 15 default events and variables are allowed to differ in the sub-group only if the difference in parameter estimation is statistically significant and significantly improves the outcome of the pseudo-log-likelihood function.

Currently the sub-groups and variables that are allowed to differ are as follows. In the Original ASEAN ex-SG group, Indonesia has different parameters than the other three economies for its intercept, stock index return, short-term interest rate, CASH/TA level and SIZE level. In the SG & HK group, Singapore and Hong Kong have different parameters for their intercept and SIGMA parameters. In the North America group, the US and Canada have a different parameter for their intercept only. In the Western Europe 1 group, the Netherlands and Belgium form a sub-group. Their parameter for their short-term interest rates is different from that of the other five economies in the group. In the Western Europe 2 group, Germany's parameter for its short-term interest rate differs from that of the other seven economies in the group.

3.4 Daily Output

Individual firms' PD: In computing the pseudo-log-likelihood functions in equation (18), only end of month data is needed. The data needs to be extended to daily values in order to produce daily PDs.

For the level variables, the last twelve end of month observations (before averaging) are combined with the current value. The current value is scaled by a fraction equal to the current day of the month divided by the number of calendar days in the month. The earliest monthly value is scaled by one minus this fraction. The sum is then divided by the number of valid monthly observations, with the current value and the earliest monthly value counting as a single observation if either or both are not missing. Not performing this scaling can lead to an artificial jump in PD at the beginning of the month. When performing the scaling, the change in level is more gradual throughout the month.

A similar procedure is done for SIGMA. Here the earliest month is not scaled, but the return from the current day to the previous month end is scaled by the square root of the fraction equal to the current day of the month divided by the number of calendar days in the month.

Computing the DTD for all firms on a daily basis using the two stage process described in Subsection 3.3 would be time consuming, even on the grid. Since there should be little change to μ , σ and δ on a day to day basis, for the daily computation of DTD these are assumed to have the same value as in the previous month's DTD calculation. In other words, the previous month's values for σ and δ together with the new day's equity value are used in equation (20) to obtain the implied asset value. This implied asset value with the previous month's values for μ , σ and δ is used in equation (22) to obtain the new day's DTD.

Aggregating PD: The CRI provides term structures of the probability distributions for the number of defaults as well as the expected number of defaults for different groups of firms. The companies are grouped by economy (using each firm's country of domicile), by sector (using the firm's Level I Bloomberg Industry Classification) and sectors within economies. With the

individual firms' PD, the expected number of defaults is trivial to compute. The algorithm used to compute the probability distribution of the number of defaults was originally reported in Anderson, Sidenius and Basu (2003). It assumes conditional independence and uses a fast recursive scheme to compute the necessary probability distribution.

Note that while this algorithm is currently used to produce the probability distribution of the number of defaults within an economy or sector, it can easily be generalized to compute loss distributions for a portfolio manager, where the exposure of the portfolio to each firm needs to be input.

Inclusion of firms in aggregation: As explained in Subsection 3.1, firms are included in an economy for calibration if the firm's primary listing is on an exchange in that economy. This is to ensure that all firms in an economy are subject to the same disclosure and accounting requirements. In contrast, a firm is included in an economy's aggregate results if the firm is domiciled in that economy. This is because users typically associate firms with their economy of domicile rather than the economy where their primary listing is, if they are different. For example, the Bank of China has its primary listing in Hong Kong, but its economy of domicile is China so the Bank of China is included in the aggregate PDs for China, and is included under China when searching for the individual PDs.

IV. EMPIRICAL ANALYSIS

This section presents an empirical analysis of the CRI outputs for the thirty economies that are currently being covered. In Subsection 4.1, an overview is given of the default parameter estimates. Subsection 4.2 explains the tests that are performed on the PDs, including accuracy ratios, the Spiegelhalter test statistic and the traffic light test. The results of the tests are also discussed.

4.1 Parameter Estimates

With 24 months of forecast horizons, two sets of parameters (default and other exits) and 12 different

groups of economies, tables of the parameter estimates occupy almost 50 pages and are not included in this Technical Report. They are available in an Annex to this report that is available via the CRI web portal. In the Annex, the parameter estimates are from calibrations performed in June 2011 using data up until the end of May 2011. As an example, plots of the default parameters for the US are given in Figures B.1 and B.2, along with the 90% confidence level. In this subsection, a brief overview is given of the general traits and patterns seen in the default parameter estimations of the economies covered by the CRI.

Recall that if a default parameter for a variable at a particular horizon is estimated to be positive (resp. negative) from maximizing the pseudo-likelihood function, then an increasing value in the associated variable will lead to an increasing (decreasing) value of the forward intensity at that horizon, which in turn means an increasing (decreasing) value for the conditional default probability at that horizon.

For the stock index one-year trailing return variable, most groups have default parameters that are slightly negative in the shorter horizons and then become positive in the longer horizons. When the equity market performs well, this is only a short-term positive for firms and in the longer-term, firms are actually more likely to default. This seemingly counter-intuitive result could be due to correlation between the market index and other firm-specific variables. For example, Duffie et al. (2009) suggested that a firm's distance-to-default (DTD) can overstate its credit-worthiness after a strong bull market. If this is the case, then the stock index return serves as a correction to the DTD levels at these points in time.

The default parameters for the short-term interest rate variable are significantly positive at one to two-year horizons for most of the groups. This is consistent with an increase in short-term interest rates signaling increased funding costs for companies in the future, increasing the probability of default. The values at shorter horizons are varied between economies from slightly negative to significantly positive, possibly indicating different lead-lag relationships between

credit conditions and the raising and cutting of short-term interest rates.

On the other hand, a sub-group of Western Europe 1 group, consisting of France, Greece, Italy, Portugal and Spain, has significantly negative default parameters associated with the short-term interest rate. This is consistent with an effect that competes with the funding costs argument: central banks typically raise interest rates to relieve inflationary pressures during expansionary periods. Thus, a high level of the short term interest rate is signaling an expansionary period where firms have a lower probability of default.

DTD is a measure of the volatility-adjusted leverage of a firm. Low or negative DTD indicates high leverage and high DTD indicates low leverage. Therefore, PD would be expected to increase with decreasing DTD. This is confirmed by the estimates, where almost all of the calibrations for the different groups lead to negative default parameters for the DTD Level, with only China's default parameter estimations hitting the constraint at zero for longer horizons.

The ratio of the sum of cash and short term investments to total assets (CASH/TA) measures liquidity of a firm. This indicates whether a company can meet its short-term obligations such as interest and principal payments. As expected, for almost all economies (Indonesia being the only exception) the default parameters for CASH/TA Level in shorter horizons are significantly negative. The magnitude of the default parameters decreases for longer horizons, confirming that CASH/TA Level is a better indicator of a firm's ability to make payments in the short term than the long term.

The ratio of net income to total assets (NI/TA) measures profitability of a firm. The relationship between PD and NI/TA is as expected: the default parameters for NI/TA Level are significantly negative for most economies and most horizons.

The logarithm of the market capitalization of a firm over the median market capitalization of firms within the economy (SIZE) does not have a consistent effect on PD across different economies. For example, in the US the default parameters for SIZE Level are

negative for shorter horizons and positive for longer horizons, suggesting that the advantages enjoyed by larger firms, such as diversified business lines and funding sources, are a benefit in the shorter term but not in the longer term. On the other hand, in Japan the default parameters for SIZE Level are negative across all horizons. These differences may reflect differences in the business environments in the respective economies.

The default parameters associated with DTD Trend, CASH/TA Trend and SIZE Trend, are negative across almost all economies and horizons. The trend variables reflect momentum. The momentum effect is a short term effect, and evidence of this is seen in the lower magnitude of the default parameters at longer horizons than at shorter horizons. The remaining trend variable is the NI/TA Trend. The current implementation of the CRI system retrieves net income only from annual financial statements. The default parameters for NI/TA Trend are constrained to be negative, but for most economies there is no clear relationship between the NI/TA Trend and the horizon. Once NI/TA from quarterly statements can be used, this will likely be more informative.

The ratio of the sum of market capitalization and total liabilities to total assets (M/B) can either indicate the market mis-valuation effect or the future growth effect. This default parameter is positive in most economies, indicating that higher M/B implies higher PD, and the market mis-valuation effect dominates. The Western Europe 2 group, consisting of Austria, Denmark, Finland, Germany, Iceland, Norway, Sweden and Switzerland, is an exception. For these economies, the associated default parameters are weakly negative at short time horizons, indicating that the future growth effect dominates.

Shumway (2001) argued that a high level of the idiosyncratic volatility (SIGMA) indicates highly variable stock returns relative to the market index, indicating highly variable cash flows. Volatile cash flows suggest a heightened PD, and this finding is consistent across all economies and most horizons, with the exception of India.

4.2 Prediction Accuracy

In-sample and out-of-sample testing: Various tests are carried out to test the prediction accuracy of the CRI PDs. These tests are conducted either in-sample or out-of-sample.

A single calibration is conducted for the in-sample tests, using data to the end of May 2011. As an example, one-year PDs are computed for Dec 31, 2000 by using the data at or before Dec 31, 2000 and the parameters from the calibration. These PDs can be compared to actual defaults that occurred at any time in 2001.

The out-of-sample analysis is done over time. The first calibration is conducted using only data up to the end of December 2000. For example, one-year PDs can be made for Dec 31, 2000 using the data at or before Dec 31, 2000 with the parameters from this first calibration. These are PDs that could have been computed at the time, since the parameters are not based on data available after that date. This process is repeated every month. That is, the second calibration is conducted using only data up to the end of January 2001, and so on.

It should be noted that for these repeated calibrations based on an expanding window of data, nothing else is changed besides the dataset. In other words, the same choice of input variables and the same choice of economy dummies within the groups are used throughout all of the calibrations.

Some of the calibration groups have too few defaults in the period before December 2000 to be able to produce stable calibration results (see Table A.8). If this is the case, the start date is advanced. Subsequently, if there are too few defaults after the start date to perform meaningful tests, only in-sample tests are performed for that calibration group. Out-of-sample tests are performed for (starting month of calibration in parentheses): China (12/2000), Japan (12/2003), India (12/2001), South Korea (12/2000), ASEAN ex-SG group (12/2000), North America group (12/2000), and Western Europe 2 group (12/2002).

Accuracy Ratio: The accuracy ratio (AR) is one of the most popular and meaningful tests of the discrimina-

tory power of a rating system (BCBS, 2005). The AR and the equivalent area under the Receiver Operating Characteristic (AUROC) are described in Duan and Shrestha (2011). In short, if defaulting firms had been assigned among the highest PD of all firms before they defaulted, then the model has discriminated well between “safe” and “distressed” firms. This leads to higher values of AR and AUROC. The range of possible AR values is in $[0, 1]$, where 0 is a completely random rating system and 1 is a perfect rating system. The range of possible AUROC values is in $[0.5, 1]$. AUROC and AR values are related by: $AR = 2 \times AUROC - 1$.

Table B.1 lists AR and AUROC values for the one-month, six-month, one-year and two-year horizons, with standard errors in parentheses. In Table B.1, both in-sample and out-of-sample results are available for calibration groups where out-of-sample testing could be performed. Other calibration groups include only in-sample results. The in-sample AR and AUROC are computed only from the starting date of the corresponding out-of-sample tests, so that the results between in-sample and out-of-sample are comparable. Only economies with more than 20 defaults entering into the AR and AUROC computation are listed. The PDs are taken to be non-overlapping. For example, the one-year AR is based on PDs computed on 31/12/2000, 31/12/2001, ..., 31/12/2010 and firms defaulting within one year of those dates, while the two-year AR is based on PDs computed on 31/12/2000, 31/12/2002, ..., 31/12/2008 and firms defaulting within two years of those dates.

The AUROC values have been provided only for the purpose of comparison with rating systems that report their results in terms of AUROC. In this report, the discussion will focus only on AR. The model is able to achieve strong AR results mostly greater than 0.80 at the one and six-month horizons for developed economies. There is a drop in AR at one and two-year horizons, but the AR are still mostly acceptable. Australia, the UK and Singapore have sharp drops in AR to below 0.60 at the two-year horizon. Hong Kong has worse than expected AR over all horizons. With Hong Kong and

Singapore, the low AR may be helped if combined with Taiwan to achieve a more stable calibration.

The AR in emerging market economies such as China, India, Indonesia, Malaysia, Philippine and Thailand are noticeably weaker than the results in the developed economies. This can be due to a number of issues. The quality of data is worse in emerging markets, in terms of availability and data errors. This may be due to lower reporting and auditing standards. Also, variable selection is likely to play a more important role in emerging markets. The variables were selected based on the predictive power in a developed economy, the US. Performing variable selections specific to the calibration group are expected to improve predictive accuracy, especially in emerging market economies. Finally, there could be structural differences in how defaults and bankruptcies occur in emerging market economies. If the judicial system is weak and there are no repercussions for default, firms may be less reluctant to default.

At horizons of one and six-months, out-of-sample AR are comparable to their in-sample counterparts. At horizons of one and two-years, out-of-sample AR can be substantially lower than the in-sample AR. This issue is more easily examined with the tests presented later.

Finally, the US has a sufficient number of financial firms and financial defaults to produce separate AR and AUROC. These are also listed in Table B.1 as out-of-sample results. The financial sector ARs are actually stronger than the non-financial sector AR. This is achieved by having only minimal differences between how financial and non-financial firms are treated.

The AR is a test of discriminatory power, or how well the rating system ranks firms in terms of credit worthiness. The following plots and tests are direct test of the PD values computed from the rating system. Each will be explained and a discussion of the results will follow.

Aggregate defaults: The time series of aggregate predicted number of defaults and actual number of defaults in each calibration group is plotted in each of the first rows of graphs in Figures B.3 to B.15. The left

graphs are for a horizon of one-month and the right graphs are for a horizon of one-year. For each month, the predicted number of defaults computed from the aggregate PD is plotted in the red line, and the actual number of defaults in the following one month (or one year) is plotted as the blue bar.

Spiegelhalter test statistic: The Brier score at time t and horizon t' is a mean square error (MSE) between the model predicted PD and the realized defaults:

$$MSE_t(t') = \frac{\sum_{i=1}^{N_t} (PD_{it}(t') - y_{it}(t'))^2}{N_t}$$

Here, N_t is the number of valid t' -horizon PDs observed at time t , $PD_{it}(t')$ is the model generated PD for firm i at time t for horizon t' . Also, $y_{it}(t')$ is the default indicator: if the i^{th} obligor defaults before $t + t'$, then $y_{it}(t') = 1$; otherwise $y_{it}(t') = 0$. The MSE statistic is small if the model generated PDs assigned to firms that eventually default is high and the PDs assigned to firms that do not default is low. In general, a low MSE indicates a good rating system. However, the best that a modeler can hope to do is to correctly estimate probabilities of default, rather than to know defaults with certainty, since defaulting is a random event (Rauhmeier and Scheule, 2005).

The Spiegelhalter test has a null hypothesis that the model has exactly predicted the actual default probabilities, and test statistic formulated as:

$$Z_t(t') = \frac{MSE_t(t') - E(MSE_t(t'))}{\sqrt{Var(MSE_t(t'))}}$$

One can compute:

$$E(MSE_t(t')) = \frac{\sum_{i=1}^{N_t} PD_{it}(t')(1 - PD_{it}(t'))}{N_t},$$

and

$$Var(MSE_t(t')) = \frac{\sum_{i=1}^{N_t} PD_{it}(t')(1 - PD_{it}(t'))(1 - 2PD_{it}(t'))^2}{N_t^2}.$$

Following Spiegelhalter (1986), under the assumption of independence among PDs and using the central limit

theorem, it can be shown that under the null hypothesis the test statistic $Z_i(t')$ approximately follows a standard normal distribution whose confidence interval can be computed routinely.

The time series of the Spiegelhalter test statistic for each calibration group is plotted in each of the second rows of graphs in Figures B.3 to B.15. The left graphs are for a horizon of one-month and the right graphs are for a horizon of one-year. The solid line is the test statistic computed at a monthly frequency, and the dashed lines are the 90% and 95% confidence intervals.

Traffic light test: Coppens, González and Winkler (2007) describe a test that is of particular relevance to regulators and parctitioners. The basic idea is to classify firms as “investment grade” by their model generated PD, observe how many of these firms end up defaulting and assess whether the actual number of defaults exceed certain confidence levels. The confidence levels are set at 80% and 99%. If the 80% confidence level is exceeded without exceeding the 99% level, the rating system is in the “amber” zone, which is allowed once every five years. Any exceedance of the 99% level places the rating system in the “red” zone.

The traffic light test results are plotted in the third rows of Figures B.3 to B.15. Following Coppens et al. (2007), the threshold for one-year PD is set at 10bps. The upper bound of the “green” zone is the green line, and the lower bound of the “red” zone is the red line. The confidence levels are computed based on the binomial distribution where all of the PDs are assumed to be 10bps. For some calibration groups, if there are too few firms with a one-year PD less than 10bps, the green line (and even the red line) will remain constant at zero. The threshold is increased by steps of 10bps until the green line is substantially different from zero.

Discussion of tests: Figure B.3 and B.4 are in-sample and out-of-sample results for the North America group. Comparing the actual versus predicted number of defaults, two points are notable. One is that for the early period in 2001 to mid-2002, the out-of sample results are very poor. One possible reason is that the

datasets for these calibration dates cover a period mainly consisting of only an economic expansion. NBER (2010) deems March 1991 to have been the trough of the business cycle, while the peak was in March 2001. Having default experience from only half a cycle would not allow the model to calibrate for different parts of the cycle. This would have played a part in variable selection at that time.

The other notable point is that the out-of-sample predicted number of defaults during the 2008-2009 financial crisis is much higher than the corresponding in-sample predictions. This could be due to the fact that because of the extraordinary government intervention at the time, the predicted defaults never occurred. As time has passed and the data window expands to include the period where financial ratios are still distressed but the default rates are relatively low, more recent calibrations from the in-sample results have modulated the predicted number of defaults in that period.

It is noteworthy that although out-of-sample ARs for the North America group are acceptable, the predicted numbers of defaults are far off in several periods. This is due to the fact that AR primarily measures how well a rating system ranks different firms looking at PD values relative to each other, and ignores the magnitude of PD values. The plots of the out-of-sample tests for the other calibration groups show a similar difference between the out-of-sample and in-sample results. The remainder of this subsection will focus on the in-sample results.

The predicted numbers of defaults are slow to capture the actual numbers of defaults in many of the economies. A possible remedy to this is to incorporate different common factors that are better leading indicators of economic distress, such as realized market volatility. Related to this lag between actual defaults and predicted defaults are the rejections of the null hypothesis in the Spiegelhalter test. For the economies that have weak AR, the difference between the predicted defaults and actual defaults is apparent.

The traffic light test is passed by all calibration groups. The “red” zone is never breached and there is never more than one violation into the “amber” zone in a period of five years.

V. PLANNED DEVELOPMENTS

The credit rating system in the CRI is meant to evolve and improve as additional features and elements are developed. Besides modifications to the current modeling framework of the forward intensity model, a change in modeling platform will be undertaken if another model proves more promising in terms of accuracy and robustness of results. In this version of the Technical Report, no platform changes are anticipated. This section describes the developments that are planned within the current modeling platform.

The existing modeling framework of the forward intensity model has proven to be robust and flexible while providing consistent term structure information on default probabilities that is difficult to obtain in different models. At the same time, much development remains to be done to improve our existing results. Areas for future development are listed below.

Default definitions: As described in Subsection 2.4, a major challenge in the implementation of a credit assessment system is in defining and classifying defaults. Different jurisdictions treat and report defaults differently, and additional work needs to be done on normalizing to a common definition of default.

Besides defining default, the search for default events is on-going. Defaults that occur on a current basis need to be captured. As well, some economies seem to have fewer defaults than would be expected so the historical default event search is also an on-going effort.

Variable selection: Up to this point, the CRI system has used the same input variables across all economies. This has been done in order to expand the coverage to a global scale as quickly as possible. Restricting to a common definition of input variables removes a bottleneck in expanding to a different geographical region.

With only a few remaining major economies to be covered (e.g. Brazil and Russia), the issue of variable selection, including both firm-specific variables and common factors, can be investigated. It is expected that removing the restriction of having commonly defined

input variables will improve the prediction accuracy, especially for emerging market economies.

In addition to new variables, the existing input variables can be fine-tuned. For example, the net income is currently taken only from annual financial reports. Once investigation is done on seasonal effects of using quarterly net income, the profitability measure will be updated in a timelier manner. Also, the idiosyncratic volatility measure uses monthly returns, and does not react quickly to sudden changes in idiosyncratic volatility. Experimentation is being conducted on using daily returns to compute this measure.

Mergers and acquisitions: Whenever there is a major merger or acquisition, the PD for the remaining company cannot be calculated until a new financial statement is released, since the merger or acquisition is reflected in the market capitalization but not in the financial statement variables. Preliminary investigations on simply combining balance sheets are promising.

Global coverage: The next region that coverage will extend to is Latin America. After which, Europe will be completed with coverage of Eastern Europe. Africa and the Middle East will remain after that, and in a final step, coverage will be extended to economies that have been missed along the way.

The RMI Credit Rating initiative is premised on the concept of credit ratings as a “public good.” Being a non-profit undertaking allows a high level of transparency and collaboration that other commercial credit rating systems cannot replicate. The research and support infrastructure is in place and researchers from around the world are invited to work on our database.

Any methodological improvements that researchers develop will be incorporated into the CRI system. In essence, the initiative operates as a “selective Wikipedia” where many can contribute but implementation control is retained.

If you have feedback on this Technical Report or wish to work with us in this endeavor, please contact us at rmicri@globalcreditreview.com.

References

- Anderson, L., J. Sidenius and S. Basu, (2003), “All your Hedges in one Basket,” *Risk*, pp. 67–72.
- Basel Committee on Banking Supervision (BCBS), (2005), “Studies on the Validation of Internal Rating Systems,” Working Paper 14.
- Basel Committee on Banking Supervision (BCBS), (2006), “International Convergence of Capital Measurement and Capital Standards — A Revised Framework.”
- Coppens, F., F. González and G. Winkler, (2007), “The Performance of Credit Rating Systems in the Assessment of Collateral used in Eurosystem Monetary Policy Operations,” European Central Bank.
- Crosbie, P. and J. Bohn, (2003), “Modeling Default Risk,” Moody’s KMV.
- Duan, J.-C., (1994), “Maximum Likelihood Estimation Using Price Data of the Derivative Contract,” *Mathematical Finance*, 4, pp. 155–167.
- Duan, J.-C., (2000), “Correction: Maximum Likelihood Estimation Using Price Data of the Derivative Contract,” *Mathematical Finance*, pp. 461–462.
- Duan, J.-C., (2010), “Clustered Defaults,” Singapore: RMI, National University of Singapore.
- Duan, J.-C., J. Sun and T. Wang, (2011), “Multiperiod Corporate Default Prediction — A Forward Intensity Approach,” Singapore: RMI, National University of Singapore.
- Duan, J.-C. and K. Shrestha, (2011), “Statistical Credit Rating Methods,” *Global Credit Review*, pp. 43–64.
- Duffie, D., A. Eckner, G. Horel and L. Saita, (2009), “Frailty Correlated Default,” *Journal of Finance*, pp. 2089–2123.
- Duffie, D., L. Saita, and K. Wang, (2007), “Multi-Period Corporate Default Prediction with Stochastic Covariates,” *Journal of Financial Economics*, 83, pp. 635–665.
- Fitch Ratings, (2011), “Definitions of Ratings and Other Forms of Opinions.”
- Merton, R.C., (1974), “On the pricing of corporate debt: The Risk Structure of Interest Rates,” *Journal of Finance*, 29, pp. 449–470.
- Monetary Authority of Singapore, (2004), “IRB Approach: Definition of Default.”
- Moody’s Investors Service, (2011), “Corporate Default and Recovery Rates, 1920–2010.”
- National Bureau of Economic Research, (2010), “US Business Cycle Expansions and Contractions.”
- Rauheimer, R. and H. Scheule, (2005), “Rating properties and their implications for Basel II capital,” *Asia Risk*, 5, pp. 51–54.
- Spiegelhalter, D., (1986), “Probabilistic prediction in patient management and clinical trials,” *Statistics in Medicine*, 5, pp. 421–433.
- Standard & Poor’s, (2011), “2010 Annual Global Corporate Default Study and Rating Transitions.”